Partial Exam

Mathematical Methods of Bioengineering
Ingenería Biomédica

20 of March 2019

The maximum time to make the exam is 2 hours. You are allowed to use a calculator and two sheets with annotations.

Problems

1. (**2 points**) Find the equation of a plane that contains the line $l(t) = (-1, 1, 2) + t(3, 2, 4)$ and is perpendicular to the plane $2x + y - 3z + 4 = 0$.

   Note: Two planes are perpendicular when their normal vector are.

2. The three-dimensional **heat equation** is the partial differential equation

$$k\Big(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\Big) = \frac{\partial T}{\partial t}$$

   (a) (**1 point**) First we examine a simplified version of the heat equation. Consider a straight wire modelled by x. Then the temperature $T(x, t)$ at time t and position x along the wire is modelled by the one-dimensional heat equation

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

   Show that the function $T(x, t) = e^{-kt}\cos x$ satisfies this equation. What happens to the temperature of the wire after a long period of time?

   (b) (**1 point**) Now show that $T(x, y, z, t) = e^{-kt}(\cos x + \cos y + \cos z)$ satisfies the three-dimensional heat equation.

3. A bioinvestigation laboratory works with cells whose shape are represented in the next figure.

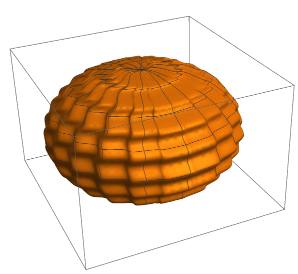

Figure 1: Cell.

While proceeding with the experiment, an unexpected incident disturbs the pressure in the essay area. The pressure at each point of the space is now given by the function

$$T(x, y, z) = xy + xz + yz$$

(a) (**2 points**) Suppose you can model the surface of each of the cells with the following equation:

$$\mathbf{x}(s,t) \equiv \begin{cases} x(s,t) = 1.5 \sin s \sin t + 0.05 \cos 20t \\ y(s,t) = 1.5 \cos s \sin t + 0.05 \cos 20s, \\ z(s,t) = \cos t \end{cases} \qquad t, s \in [-\pi, \pi]$$

Compute the variation of the pressure on the surface when $s = \frac{\pi}{2}$ and $t = \frac{\pi}{2}$.

(b) (**1 point**) Suppose a microorganism is at the point $(-1, 0, 0)$. In which direction should the cell move in order to keep pressure constant? Explain your answer.

4. A laboratory is working in a **nanotechnology** experiment that is trying to model a new prototype of carbon nanotube as shown in figure 2. The surface in nanometers (nm) is given by the equation

$$z = 2(x^2 + y^2)e^{-x^2-y^2}$$

(a) (**1 point**) Find the critical points of the nanotube.

(b) (**1 point**) Is the origin a minimum/maximum? Explain your answer.

(c) (**1 point**) Write the equation in cylindrical coordinates. Which variables appear on the equation? Does the equation represent the same figure when $\theta = 0$ and $\theta = \pi/2$?

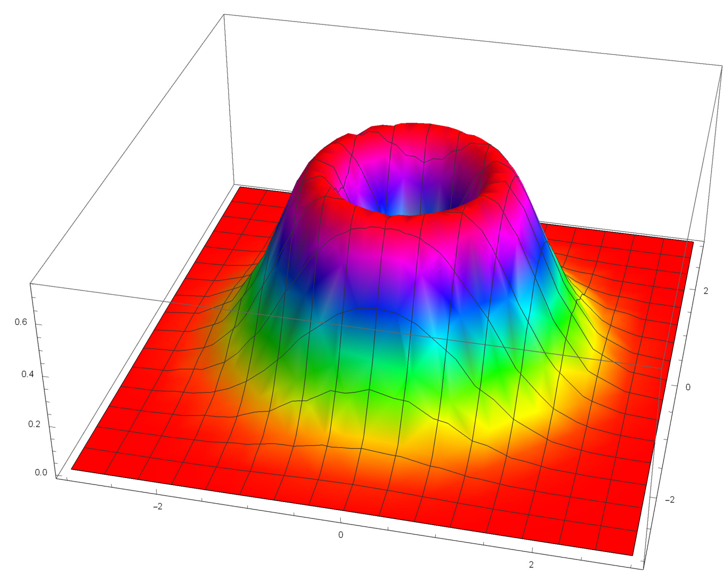

Figure 2: Representation of the prototype.